Relating Watershed Physical Characteristics to Instantaneous Unit Hydrograph Parameters for Rainfall-Runoff Modeling in Central Texas

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- Lamar University
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Outline

- Background and Significance
- Study Area
- Data Base
- Methodology:
  - Regional Unit Hydrographs by Regression
  - Regional Unit Hydrographs by Time-area-method
- Results
- Conclusions
Background and Significance

- **Hydraulics and Transportation Goals**
  - Maintain service up to $n$-year event.
  - Survive extreme events.
  - Reasonable economics.
Maintain Service

~ 4% chance (25 year) discharge
Survive Extreme Events

~ 1% chance (100 year) discharge minimal damage.
Reasonable Economics

Costs need to be compared against risk of failure and traffic load
Research Scope

- Evaluate/develop regionalized unit hydrograph methods for Texas watersheds in the 200-acre to 10 square mile range.
- Application is watersheds with limited/no gage data for use by the Texas Department of Transportation.
- Currently the department uses the NRCS unit hydrograph as implemented in HEC-HMS.
Study Area

90 watersheds
Database(s)

- Precipitation
  - Texas, Louisiana, Oklahoma, New Mexico
- Paired rainfall-runoff (1600 events)
  - 1600 events
  - 90 watersheds in Texas
- Watershed characteristics
  - 90 watersheds in Texas
Texas Hyteograph(s)

- Link to Will’s Presentation
Basin Characteristics

- Descriptive (Land use, % impervious, etc.)
  - Compiled by U.S.G.S. using GIS

- Physical
  - Compiled manually by University of Houston
  - Compiled by the U.S.G.S. using GIS.
    - The two approaches produced practically identical results for the common characteristics
    - All correlations are based on the U.S.G.S. physical characteristics.

- Of the 90+ watersheds
  - 58 are smaller than 10 square miles in drainage area
  - 72 are smaller than 20 square miles in drainage area
## Basin Characteristics

### Explanatory Variables:

| Module          | Station | SubBasin | Total drainage area (mi²) | Basin length (mi) | Basin perimeter (mi) | Average basin slope (ft/mi) | Basin relief (ft) | Minimum basin elevation (ft) | Maximum basin elevation (ft) | Average basin elevation (ft) | Headwater elevation (ft) | Pourpoint (outlet) elevation (ft) | Effective basin width (mi) | Basin shape factor | Elongation ratio | Rotundity of basin | Compactness ratio | Relative relief (ft/mi) | Basin factor (MCL²/A) | Main channel length (mi) | Main channel slope (ft/mi) | Main channel sinuosity ratio | Slope ratio of main channel to basin slope | Alternate Main channel slope |
|-----------------|---------|----------|---------------------------|------------------|----------------------|----------------------------|-------------------|----------------------------|-----------------------------|----------------------------|-------------------|--------------------------------|--------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|--------------------------------|---------------------------|-----------------------------|-----------------------------|-----------------------------|
| Austin BartonCreek none 08155200 | 69.6 | 18.2 | 67.7 | 372.3 | 752.2 | 750 | 1503 | 1479 | 750 | 4.9 | 3.70 | 0.53 | 2.91 | 0.05 | 25.6 |
| Austin BartonCreek none 08155300 | 94.5 | 24.4 | 81.2 | 415.4 | 868 | 866 | 1732 | 1703 | 865 | 5.0 | 4.70 | 0.68 | 3.70 | 0.07 | 21.3 |
| Austin BearCreek none 08158810 | 12.3 | 5.0 | 19.7 | 318.8 | 374.1 | 866 | 1242 | 1053 | 868 | 5.4 | 5.70 | 0.71 | 3.87 | 0.06 | 20.3 |
| Austin BearCreek none 08158825 | 21.0 | 8.8 | 29.1 | 436.2 | 500 | 498 | 1001 | 891 | 494 | 6.2 | 5.90 | 0.68 | 4.17 | 0.05 | 19.3 |
| Austin BoggyCreek none 08158050 | 12.6 | 6.0 | 20.6 | 179.9 | 209.9 | 567 | 737 | 429 | 737 | 4.9 | 4.10 | 0.53 | 3.20 | 0.04 | 18.3 |
| Austin BoggySouthCreek none 08158880 | 9.8 | 3.8 | 12.5 | 202.0 | 245.7 | 567 | 737 | 429 | 737 | 4.9 | 4.10 | 0.53 | 3.20 | 0.04 | 18.3 |
| Austin Bull Creek none 08154700 | 22.8 | 8.5 | 31.6 | 191.5 | 235.2 | 567 | 737 | 429 | 737 | 4.9 | 4.10 | 0.53 | 3.20 | 0.04 | 18.3 |
| Austin Little Walnut Creek none 08158380 | 5.3 | 3.4 | 13.0 | 155.8 | 192.4 | 567 | 737 | 429 | 737 | 4.9 | 4.10 | 0.53 | 3.20 | 0.04 | 18.3 |
| Austin Onion Creek none 08158800 | 12.7 | 20.8 | 7.8 | 330.6 | 374.1 | 866 | 1242 | 1053 | 868 | 5.4 | 5.70 | 0.71 | 3.87 | 0.06 | 20.3 |
| Austin Onion Creek none 08158825 | 21.0 | 8.8 | 29.1 | 436.2 | 500 | 498 | 1001 | 891 | 494 | 6.2 | 5.90 | 0.68 | 4.17 | 0.05 | 19.3 |
| Austin Shoal Creek none 08156650 | 2.7 | 2.1 | 10.2 | 160.1 | 193.2 | 567 | 737 | 429 | 737 | 4.9 | 4.10 | 0.53 | 3.20 | 0.04 | 18.3 |
| Austin Shoal Creek none 08156700 | 6.3 | 3.8 | 15.2 | 165.4 | 242.0 | 671 | 737 | 429 | 737 | 4.9 | 4.10 | 0.53 | 3.20 | 0.04 | 18.3 |
| Austin Waller Creek none 08157000 | 2.2 | 3.8 | 10.1 | 125.2 | 212.6 | 560 | 737 | 429 | 737 | 4.9 | 4.10 | 0.53 | 3.20 | 0.04 | 18.3 |
| Austin Waller Creek none 08157500 | 4.2 | 4.7 | 14.8 | 145.3 | 192.4 | 567 | 737 | 429 | 737 | 4.9 | 4.10 | 0.53 | 3.20 | 0.04 | 18.3 |
| Austin Walnut Creek none 08158600 | 26.4 | 8.0 | 32.9 | 193.0 | 401.7 | 566 | 966 | 775 | 945 | 566 | 3.3 | 2.42 | 0.73 | 1.90 | 0.10 | 35.0 |
| Austin Walnut Creek none 08158400 | 5.7 | 4.0 | 14.2 | 126.9 | 167.1 | 634 | 801 | 570 | 801 | 570 | 3.4 | 1.10 | 1.08 | 0.87 | 0.07 | 26.6 |
| Austin Walnut Creek none 08158500 | 12.1 | 7.2 | 22.2 | 184.0 | 204.0 | 466 | 607 | 506 | 607 | 506 | 3.4 | 1.10 | 1.08 | 0.87 | 0.07 | 26.6 |
| Austin Walnut Creek none 08158600 | 53.6 | 14.4 | 53.7 | 215.5 | 528.9 | 439 | 968 | 702 | 968 | 439 | 3.7 | 3.87 | 0.57 | 3.04 | 0.12 | 26.6 |
| Austin West Boulder Creek none 08155550 | 2.7 | 3.0 | 10.2 | 235.4 | 243.2 | 461 | 704 | 616 | 704 | 616 | 4.8 | 0.52 | 0.92 | 0.02 | 26.6 |
| Austin Wilbarger Creek none 08159150 | 4.5 | 2.9 | 12.1 | 107.2 | 169.9 | 673 | 843 | 575 | 843 | 575 | 3.1 | 1.35 | 1.35 | 0.12 | 26.6 |
| Austin Williamson Creek none 08158620 | 6.3 | 4.4 | 14.8 | 336.3 | 371.5 | 801 | 1116 | 959 | 1116 | 959 | 4.4 | 0.65 | 1.07 | 1.23 | 0.09 | 26.6 |
| Austin Williamson Creek none 08156930 | 18.7 | 9.5 | 30.3 | 217.0 | 492.6 | 623 | 1116 | 839 | 1116 | 839 | 6.2 | 2.0 | 0.81 | 1.62 | 0.10 | 26.6 |
| Austin Williamson Creek none 08156970 | 27.4 | 15.8 | 42.0 | 207.4 | 607.4 | 509 | 1116 | 776 | 1116 | 776 | 5.0 | 2.0 | 0.98 | 1.55 | 0.12 | 26.6 |
| Dallas Ash Creek none 08057520 | 7.2 | 8.4 | 16.9 | 116.9 | 174.5 | 439 | 606 | 525 | 606 | 525 | 4.3 | 1.42 | 0.56 | 3.24 | 1.05 | 4.99 | 5.44 | 1.00 | 0.30 | 26.6 |
Unit Hydrograph Analysis

- The instantaneous unit hydrograph (IUH) is a direct runoff hydrograph (DRH) resulting from a unit depth of an effective precipitation hyetograph (EPH) applied uniformly over a watershed.

- A major advantage of an IUH over a unit hydrograph is that the IUH does not require the effective precipitation hyetograph to have a specific duration.
IUH Concept

Outlet

Unit Depth at Time < 0;
Discharge at Time < 0

Unit Depth at Time = 0;
Discharge at Time = 0

Remaining Depth at Time > 0;
Discharge at Time > 0

Remaining Depth

Cumulative Discharge

Discharge (Rate)

Time
Instantaneous Unit Hydrographs

The direct runoff hydrograph is computed as the convolution of the effective precipitation hyetograph and the IUH kernel function.

\[
Q(t) = \int_0^t i(\tau)u(t - \tau) d\tau
\]  

[Eqn. 1]

\(i(t)\) is the EPH (precipitation rate as a function of time)

\(u(t)\) is the IUH (unit response rate as a function of time)

\(Q(t)\) is the DRH (direct runoff rate as a function of time).
Model Hydrographs

- Modeled the conversion of precipitation to runoff as a hybrid of a translation hydrograph, and a series (cascade) of storage elements.

\[
Q(t)_m = \int_{0}^{t} \{i(t)\} A \frac{2}{\bar{t}} \frac{1}{\Gamma(N)} \left( \frac{(t - \tau)^{2N-1}}{\bar{t}^{2N-1}} \right) \exp\left(-\left(\frac{t - \tau}{\bar{t}}\right)^2\right) d\tau
\]

\[
\{i(t)\} = 0 \quad \text{if} \quad \int_{0}^{t} p(\tau) d\tau \leq I_a
\]

\[
\{i(t)\} = C_r p(t) \quad \text{if} \quad \int_{0}^{t} p(\tau) d\tau > I_a
\]
Finding Model Parameters

- Calculate DRH from the effective rainfall signal (Equation 2) and adjust values until some merit function is minimized.

\[ SSE = \sum_{i=1}^{NOBS} (Q_s - Q_o)_i^2 \]

\[ Q_p \text{MAD} = \left| Q_s(t_{\text{peak}}) - Q_o(t_{\text{peak}}) \right| \]

- A modified direct-search technique (Hooke and Jeeves, 1961). (Essentially a grid-search – many simulations)
- Used a cluster computer to perform the computations
“Fitted” Hydrographs

iuh2_sta08057320_1973_0603.dat

iuh2_sta08057320_1973_0603.dat
Regional Regression Equations

Regionalization:

- Regression Models using Watershed Characteristics
  - Use power-law model values of distribution and loss model parameters.
  - Least squares fit to median values for each watershed.

\[
\begin{align*}
I_a &= 0 \\
C_r &= 0.137 A^{-0.109} S^{0.206} \\
\bar{t} &= 138 \left( \frac{A}{P} \right)^{0.334} S^{-0.500} \\
N &= 2.43 P^{0.102} S^{0.064}
\end{align*}
\]
Regional Regression Equations
Regional Regression Equations
Performance Testing

~2 sq.mi.
Performance Testing

~12 sq.mi.
Finding Model Parameters

- Time-area-analysis.
  - Relates travel time to different areas in the watershed.
  - Also called Contributing Area Method.

- Procedure:
  - Create a histogram of contributing area versus time.
  - Apply precipitation hyetograph, and use the histogram as the kernel function in the convolution integral.
  - Result of convolution is a direct runoff hydrograph.
Contributing Area Histogram

Watershed Map

Plot is the time-area histogram

Area between travel time intervals

All water on this isochrone takes 10 time units to reach outlet

All water on this isochrone takes 5 time units to reach outlet

(c) Time-area histogram
Synthetic Unit Graph

Convolve the Precipitation and Time-Area Histogram

\[ Q(t) = \int_0^t I(\tau)u(t-\tau)d\tau, \]

(d) Hydrograph at outlet
Determining the Time-Area Diagram

- One needs to have a good idea of the path (trajectory) a water particle will follow.
- One needs to know how fast (velocity) the particle will move at different points along that path.
Determining the Time-Area Diagram

Instead of focusing on large distances and areas, we divide the watershed into small areas and distances.

Notice the fundamental difference in the subdivision as compared to the classical isochrone division.
Determining the Time-Area Diagram

- Place “particles” in each subdivision.
- Specify particle kinematics (how the particles will move).
- Allow the particles to move following the kinematics equations.
Determining the Time-Area Diagram

- Track particles over time.
  - Recover path lines (if we are interested)
- Accumulate arrival times at the outlet.
  - Time series of cumulative arrivals (normalize by total particle arrivals)
Particle Kinematics

Trajectory equations:

\[
\begin{align*}
\frac{dx_p}{dt} &= u_p(x_p(t), y_p(t), t) \\
\frac{dy_p}{dt} &= v_p(x_p(t), y_p(t), t)
\end{align*}
\]

Euler-difference form (how we actually calculate):

\[
\begin{align*}
x_p(t + \Delta t) &= x_p(t) + u_p(x_p(t), y_p(t), t)\Delta t \\
y_p(t + \Delta t) &= y_p(t) + v_p(x_p(t), y_p(t), t)\Delta t
\end{align*}
\]
Velocity Models

Like other methods, the difficulty in specifying velocity field.

- Flow Potential: Velocity is proportional to some potential.
- Newtonian mechanics: Momentum equation for each particle.

Models (pathline coordinates)

- Linear
  \[ u(\xi) = k \cdot \frac{dz}{d\xi} \]

- Quadratic
  \[ u(\xi) \cdot u(\xi) = k^2 \cdot \frac{dz}{d\xi} \]

- Newtonian (concept)
  \[ m \frac{du_p}{dt} = -mg \cdot \frac{dz}{dx} - mk^2 u(x, y) \cdot u(x, y) - d_f \left( \frac{dp}{dx} \right) \]
Unit Velocity Coefficient

The “k” term represents velocity magnitude at unit slope (1:1) and this term should be different with surface type, etc.

In the quadratic model, we choose a Manning’s type structure for $k$.

$$k^2 = \left(\frac{1.5}{n} \frac{d^2}{d^3}\right)^2$$

$$u(\xi) = \frac{1.5}{n} d^3 \left(\frac{dz}{d\xi}\right)^{\frac{1}{2}}$$

Key point is that the localized watershed slope conveys the drainage information.
Direction and Slope

- **8-cell pour-point-model.**
  - 8 elevation differences.
  - Largest “downhill” difference is local direction.
  - Slope is ratio of difference in elevation and distance to adjacent cell center in local direction.
  - Velocity from path-line equation.
Watershed Elevation Maps

Require information about the spatial distribution of watershed elevation.

- Manually from USGS topographical (paper) maps
- engineering survey
- USGS digital elevation maps.

The representation is a grid whose horizontal and vertical positions represent locations on the surface of the Earth, and whose values represent elevation above some datum.

The numerical experiments presented below are all based on USGS 30-meter DEM maps downloaded from the Internet.
Study Area

Same as before.

- Digital Elevation Maps
- Digital Raster Graphics
Preparing the Model

Locate the watershed.
Manually or automatically delineate the watershed boundary.
Prepare a DEM for the area
Locate the watershed on the DEM, and extract the elevations.
This elevation array is input to the a particle tracking code.

Ash Creek Sta08057320
Tracking the Particles

Large computational burden – each particle must be followed from beginning to outlet and displacements and elapsed time recorded.

Paths are independent in potential based models (one particle at a time).

Paths are interdependent in Newtonian model (all particles at once).

NASDC/S2PC cluster

Share an NFS mount over public internet to have 24 CPUs all operating on the same data.
Numerical Experiments

Initial Particle Array for Ash Creek

- Particles are placed uniformly over watershed.
- Coarse grid
  - 150 meter resolution
Running the Model

Particle Array at 5 minutes simulation time (Yellow)

Particles are concentrating towards lower elevations.
Running the Model

Particle Array at 15 minutes simulation time (Green)

Particles are concentrating toward lower elevations.

Tributaries apparent.
Running the Model

Particle Array at 30 minutes simulation time (Blue)

Particles are concentrating toward lower elevations.

Tributaries more apparent.
Running the Model

5 minutes (Maps are at different resolutions)
Running the Model

15 minutes
Running the Model

30 minutes
Empirical Cumulative IUH

Conceptually this is a time-area histogram.
Curvilinear IUH

Fit an IUH function to the Empirical IUH
Recover parameter values for the curvilinear IUH and use this as the SUH for modeling runoff using actual precipitation.
Sample Results

- Entire database has been modeled once using $k =0.04 \ d=0.2$.

  These numerical values are from a trial-and-error calibration to a single storm on the Ash Creek watershed.

- Entire set of storms (1600+) is recreated on the 90 watersheds using the runoff coefficients from the statistical analysis.
Ash Creek

Storm 1

- Dallas Module, 6.92 square miles

Storm 2
Rush Branch

DEM rendering

Dallas Module, 1.22 square miles.

Empirical Cumulative IUH

IUH from PT model
Rush Branch

Dallas Module, 1.22 square miles.
Slaughter Creek

DEM Rendering

Austin Module, 8.24 square miles

IUH from PT Model
Slaughter Creek

**Storm 1**

- Austin Module, 8.24 square miles

**Storm 2**
Austin Module

Figure 1. Map of the 29 study watersheds in the Austin module overlain in a mosaic of digital raster graphics (DRG) maps derived from U.S.G.S. 1:24000 series topographic quadrangle maps. Horizontal and vertical scales are meters Easting and Northing in UTM coordinates.
Figure 2. Map of the 20 study watersheds in the Austin module overlain on a shaded relief mosaic derived from U.S.G.S. 30-meter digital elevation maps (DEM). Horizontal and vertical scales are meters Easting and Northing in UTM coordinates.
Barton Creek

- **Austin Module**
- ~ 89 sq.mi.
Dallas Module

Figure 1. Map of the 21 study watersheds in the Dallas module overlain on a mosaic of digital raster graphics (DRG) maps derived from U.S. G.S. 1:24000 series topographic quadrangle maps. Horizontal and vertical scales are meters Easting and Northing in UTM coordinates.

Figure 2. Map of the 21 study watersheds in the Dallas module overlain on a shaded relief mosaic derived from U.S.G.S. 30-meter digital elevation maps (DEM). Horizontal and vertical scales are meters Easting and Northing in UTM coordinates.
Woody Branch

- Dallas Module
- ~11 sq.mi.
Fort Worth Module

Figure 1. Map of the 8 study watersheds in the Fort Worth Module overlain on a mosaic of digital raster graphics (DRG) maps derived from U.S.G.S. 1:24000 series topographic quadrangle maps. Horizontal and vertical scales are meters Easting and Northing in UTM coordinates.

Figure 2. Map of the 8 study watersheds in the Fort Worth Module overlain on a mosaic of shaded relief maps derived from U.S.G.S. 30-meter digital elevation maps (DEM). Horizontal and vertical scales are meters Easting and Northing in UTM coordinates.
Sycamore Creek

- Fort Worth
- ~1 sq. mi.
San Antonio Module

Figure 1. Map of the 13 study watersheds in the San Antonio Module overlain on a mosaic of digital raster graphics (DRG) maps derived from U.S.G.S. 1:20000 series topographic quadrangle maps. Horizontal and vertical scales are meters Easting and Northing in UTM coordinates.

Figure 2. Map of the 13 study watersheds in the San Antonio module overlain on a shaded relief mosaic derived from U.S.G.S. 30-meter digital elevation maps (DEM). Horizontal and vertical scales are meters Easting and Northing in UTM coordinates.
Small Rural Module

- 10 maps to show all watersheds
- Wide geographic distribution
- Similar in appearance to the urban clusters
Figure 1. Map of 5 study watersheds in Small Rural (Colorado/Deep) and Shallowwater module overlain on a mosaic of digital coverages (DRG) maps derived from U.S.G.S, 1:24,000 series topographic maps. Horizontal and vertical scales are meters Easting and Northing in UTM coordinates.

Figure 2. Map of 5 study watersheds in Small Rural (Colorado/Deep and Shallowwater) module overlain on shaded relief mosaic derived from U.S.G.S, 30-meter digital elevation maps (DEM). Horizontal and vertical scales are meters Easting and Northing in UTM coordinates.
Colorado/Deep

- Small Rural
- ~ 5 sq.mi.
Figure 1. Map of 2 study watersheds in Small Rural (Brazos/Pond-Elm) module overlain on a mosaic of digital raster graphics (DRG) maps derived from U.S.G.S. 1:24000 series topographic quadrangle maps. Horizontal and vertical scales are meters Easting and Northing in UTM coordinates.

Figure 2. Map of 2 study watersheds in Small Rural module (Brazos/Pond-Elm) overlain on a shaded relief mosaic derived from U.S.G.S. 30-meter digital elevation maps (DEM). Horizontal and vertical scales are meters Easting and Northing in UTM coordinates.
Brazos/Pond-Elm

- Small Rural
- ~23 sq.mi.
Summary and Conclusions

- **Statistical methods (regression models).**
  - Produces qualitatively reasonable results based on three simple to determine physical characteristics
    - (the method can be performed manually).
  - The approach is compatible with current modeling methodology after some variable transformations (not presented) and offers the ability to compare performance with actual events.

- **Time-area using particle tracking** an alternate approach to produce a unit hydrograph model.
  - Examples using quadratic model, single calibration by trial and error on Ash Creek and same values used for all other watersheds.
  - Demonstrates that a significant component of watershed response is explained entirely by watershed topography.
Summary and Conclusions

- Time-area (continued)
  - Will incorporate more complex physics at significant computation cost.
  - NOT FAST. It takes a long time to track the particles.
  - Feasible on purpose-built cluster computer.
- Fails at about 100 sq.mi.
  - Unknown if this is a computational issue, or a fundamental limitation.
Project Status

- 0-4193 is completed. Several research products (reports) published by U.S.G.S.
  - 1 year implementation project to train selected design engineers how to use the distribution models.

- 0-4194 is near completion. Final report around late August.
  - 1 year modification to examine infiltration capacity models as alternate rainfall-loss.

- 0-4696 is near completion. Final report around late August.
Future Directions

- **Statistical models**
  - Addition of improved loss model based on soil physics (infiltration capacity approaches).
  - Increase rainfall-runoff database to include more East Texas watersheds (Western Louisiana would be welcomed too!)

- **Particle tracking methodology**
  - Extension to Newtonian model, test on all watersheds (removes the pit handling algorithm common to all current terrain-based hydrologic models).
Find out more

http://cleveland1.cive.uh.edu/webdocs/Texas_Hydrology/texas_hydrology.html
- Databases are located at above URL.
- Published reports (as .PDF).
- Links to the other research teams.
- “How-to” documents for:
  - Getting DRG/DEMs and preparing the ASCII grid without a GIS.
  - Building a cluster computer for these kinds of problems.